Modeling of the DEF-ALFA Autonomous Underwater Vehicle (AUV)

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Abstract

This article presents a modeling of the dynamics of the DEF-ALFA autonomous underwater vehicle designed within the search project PID UNDEFI 484/2019 of the National Defense University. This paper proposes a previous study of the general characteristics of AUVs regarding their dynamic behavior and proposes a working model to apply it in the DEF-ALFA. On the basis of the developed model, the parameters of the vehicle were obtained and a simulation was carried out, allowing the realization of future studies to evaluate the performance of the DEF-ALFA and propose improvements.

I. INTRODUCTION

1. Fundamentals

An autonomous underwater vehicle is a device with a propulsion system and a control system that allow it to move through water in three dimensions, with the ability to follow pre-programmed trajectories. The vast majority of these vehicles are equipped with on-board sensors, which makes it possible to measure different ocean parameters, referencing them both spatially and temporally. They can be programmed to navigate at a constant pressure or depth or vary their depth autonomously. They have a high data collection capacity with high frequency sampling, which makes them highly productive.

One of the great advantages of these underwater vehicles is their autonomy, since they contain their own energy source, generally based on rechargeable batteries, which allows them to work continuously. By not requiring a line of communication between the vehicle and the surface, it is possible to minimize communication problems, which are of great importance in the aquatic environment. These properties make them useful in scientific exploration tasks, oceanographic sampling, and underwater exploration, being in many cases the best option for some tasks, since they do not suffer from the limitations imposed by cables, as in the case of underwater robots.

Obtaining the dynamic model of an autonomous underwater vehicle makes it possible to design and implement control strategies and navigation systems that make it possible to carry out different programmed missions autonomously. For the dynamic modeling of these vehicles, the determination of coefficients that allow expressing the linear and non-linear relationships of the forces and moments that act on the vehicle is required.

Although the analytical and semi-empirical methods used to obtain models based on physical principles make it possible to determine the vast majority of model parameters, not all coefficients can be determined analytically. That is why these methods need to be combined with other techniques, in order to find the numerical values of some of those parameters.

2. -Kinematics of an autonomous underwater vehicle

A mathematical analysis based on the static and dynamic behavior of an autonomous underwater vehicle AUV (for the acronym in English of Autonomous Underwater Vehicle), allows simulating and controlling the behavior of these marine vehicles in the water. In the following study, the mobile under study is considered as a rigid solid, this assumption allows us not to take into account the forces that act in a specific way, between the mass elements of the vehicle, for its dynamic analysis. To obtain the equations that govern the movement of a rigid body, it is necessary to define an inertial reference system. In this case, the Earth is taken as the inertial system reference, assuming that the acceleration of a point on the Earth's surface, due to its rotation, can be neglected in the case of these underwater vehicles. This approximation is valid, in this situation, since the movement of the Earth has little effect on marine vehicles that move at low speed, such as AUVs) [1]. According to these considerations, the inertial reference system originating from an OT point in solidarity with the Earth is defined, where the X axis points to the north, the Y axis to the east and the Z axis to the center of the Earth.

If an AUV is considered as a rigid body with six degrees of freedom 6(DOF), where these six degrees of freedom are determined by the independent displacements and rotations of the vehicle [3], three coordinates will be necessary to determine its position and three to know its orientation. In this way, the first three coordinates describe the position and linear movement of the vehicle, and the other three coordinates make it possible to determine its orientation and rotary movement.

Normally in underwater vehicles, linear and angular velocities are associated with a mobile coordinate system located in the vehicle and their time derivatives are measured with respect to the reference frame of the body. Thus, it is useful to define a coordinate system associated with the AUV, originating from a point belonging to the vehicle. Thus, the coordinate system $A = [\vec{\mathbf{x}}_A, \vec{\mathbf{y}}_A, \vec{\mathbf{z}}_A]$ is defined in solidarity with the AUV, with origin at its center of mass (OA), where the axes x_A; y_A and z_A are made to coincide with the axes of inertia of the AUV, which facilitates dynamic analysis. The axis $\vec{\mathbf{x}}_A$ is taken coincident with the direction of advance of the AUV, ,$\vec{\mathbf{y}}_A$ is orthogonal to $\vec{\mathbf{x}}_A$ and is positive towards starboard in the horizontal plane, while $\vec{\mathbf{z}}_A$ is oriented downward and orthogonal to the $\vec{\mathbf{x}}_A$,$\vec{\mathbf{y}}_A$ plane, as shown in Figure 1.

To study the position, velocity and acceleration of the vehicle, it is necessary to convert the parameters.

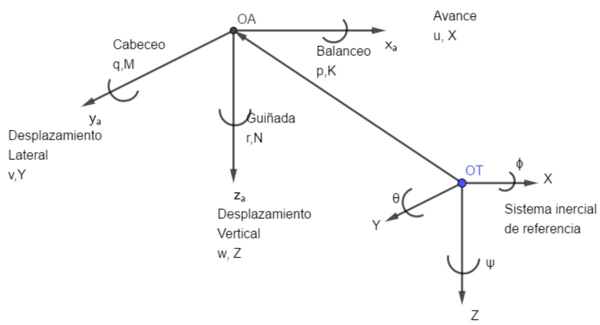

Figura 1

2.1 Kinematic equations

The kinematic equations can be expressed in vectorial form using the vectorial representation proposed by Fossen [1] and Antonelli [2] among others, for the approach of linear and non-linear equations, which describe the dynamics of the AUV. The six components of position and attitude of the AUV, which describe the movement of a marine vehicle in the six degrees of freedom 6 DOF, referred to the inertial frame, are:

η = [x, y, z, φ, θ, ψ] ^{T}

$$\eta = \begin{bmatrix}\eta_1 \\ \eta_2\end{bmatrix} \quad \text{con } \eta_1 = [x,y,z]^T \quad y \quad \eta_2 = [\varphi, \theta, \psi]^T \tag{1}$$

where η is the position and orientation vector with coordinates in the inertial reference system. The vector η_1 makes it possible to determine the position of the vehicle with respect to the fixed ground system and η_2 determines its orientation with respect to it, given by the Euler angles (φ, θ, ψ). [3]

The movements of the AUV referred to the frame fixed to the body of the vehicle, are thus defined by the six velocity components, as indicated in Figure 1:

υ = [u, v, w, p, q, r] ^{T}

being: u advance (surge), v sway (sway), w vertical displacement (heaven), p roll (roll), q pitch (pitch), and r (yaw),

Then the velocity of the AUV, in coordinates of the body's frame of reference, can be represented as:

$$\upsilon = \begin{bmatrix}\upsilon_1 \\ \upsilon_2\end{bmatrix} \text{, con } \upsilon_1 = [u \quad v \quad w]^T \; y \quad \upsilon_2 = [p \quad q \quad r]^T \tag{2}$$

where υ_1 is the linear velocity of the vehicle, measured in body frame coordinates, and υ_2 represents the angular velocity of the AUV, measured in the frame of reference attached to the vehicle. From now on, the notation ω will be used for the vector υ_2, in such a way that: $\omega = [p \quad q \quad r]^T$.

The notation used for each variable according to SNAME (1950) [6] can be summarized in Table 1.

GDL		Forces / Moments	Linear and angular velocity	Position and angle of Euler
1	Movement in x-axis direction	X	u	x
2	Movement in the y-axis direction	Y	v	y
3	Movement in the y-axis direction	Z	w	z
4	Rotation in axial x axis	K	p	ϕ
5	Rotation in axial y axis	M	q	θ
6	Rotation in axial z axis	N	r	ψ

Table 1: Notation used for each variable according to SNAME (1950)

These components are relative to a coordinate system moving with the current of the liquid where the vehicle is submerged. If υ is considered as the relative speed of the axles attached to the vehicle with respect to the flow in which it moves, and it can be assumed that the axles move at a speed υc due to the current, with these considerations the speed of the vehicle could be expressed. AUV with respect to inertial axes, such as:

$$\dot{\eta} = d(\eta)/d(t) \quad \text{with} \quad \dot{\eta} = J\eta(\upsilon + \upsilon_c) \tag{3}$$

where, $J\eta$ is the rotation matrix between both axes [1].

2.2 Rotation Matrix

To study the position, velocity and acceleration of the vehicle in both frames of reference, the Euler angle transformation is used: roll (ϕ), pitch (θ), yaw (ψ) [5]. Euler established that two independent orthonormal coordinate frames (with a common origin) can be related by a succession of no more than three rotations about the coordinate axes. This means that if the sequence of axes to be rotated is known, only three Euler angles are needed to fully define the total rotation. Using Euler's theorem, it is possible to see a sequence of rotations about different axes as a single rotation around an axis, since each rotation, having a matrix associated with it, means that the sequence of rotations has, in turn, a single matrix associated with it. . These matrices describe the mutual orientation between the two coordinate systems and their column vectors are the direction cosines of the axes of one coordinate system with respect to another. Using this property for the three Euler angles φ, θ, and y ψ, which allow determining the orientation of the vehicle with respect to the inertial frame.

Carrying out this sequence of rotations for each plane that determines each pair of orthogonal axes, the fundamental rotation matrices are obtained. [3].

$$R(x,\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C_\phi & -S_\phi \\ 0 & S_\phi & C_\phi \end{bmatrix};\; R(y,\theta) = \begin{bmatrix} C_\theta & 0 & S_\theta \\ 0 & 1 & 0 \\ -S_\theta & 0 & C_\theta \end{bmatrix};\; R(z,\psi) = \begin{bmatrix} C_\psi & -S_\psi & 0 \\ S_\psi & C_\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Where S_x=sen (x) y C_x=cos(x).

The matrix representing these rotations is mathematically equivalent to: $R(\varphi, \theta, \psi) = R(x, \varphi)\, R(y, \theta)\, R(z, \psi)$. Now, the rotation matrix $R(\varphi, \theta, \psi)$ is an orthogonal matrix, therefore $R^{-1}= R^T$, which implies that the same result is obtained by transforming a vector of the frame of reference fixed to the body to inertial frame, reversing the order of rotation. Which is mathematically equivalent to performing the following product, $R = Rz,\psi\; Ry,\theta\; Rx,\varphi$. This is how the transformation R is expressed, which allows relating the linear velocity vector in an inertial reference frame to the reference frame of the body and the rotation matrix of the inertial structure with respect to the mobile reference system located in the body, can be expressed as:

$$R = \begin{bmatrix} C_\psi C_\theta & C_\psi S_\theta S_\phi - S_\psi C_\phi & C_\psi S_\theta C_\phi + S_\psi S_\phi \\ S_\psi C_\theta & S_\psi S_\theta S_\phi + C_\psi C_\phi & S_\psi S_\theta C_\phi - C_\psi S_\phi \\ -S_\theta & C_\theta S_\phi & C_\theta C_\phi \end{bmatrix} \quad (4)$$

$$R^T = \begin{bmatrix} C_\psi C_\theta & S_\psi C_\theta & -S_\theta \\ C_\psi S_\theta S_\phi - S_\psi C_\phi & S_\psi S_\theta S_\phi + C_\psi C_\phi & C_\theta S_\phi \\ C_\psi S_\theta C_\phi + S_\psi S_\phi & S_\psi S_\theta C_\phi - C_\psi S_\phi & C_\theta C_\phi \end{bmatrix} \quad (5)$$

Once the rotation matrix is known, the kinematic rotation equations can be established, which allow determining the relationships between the angular velocities of the AUV expressed in both reference axes. Relating the derivative of the orthonormal matrix and its orthonormality property, the angular velocity vector, with respect to the coordinate axes of the body, is related to the generalized velocities $(\dot{\phi},\dot{\theta},\dot{\psi})$, where the Euler angles are valid. Using the Wη transformation matrix

$$W\eta = \begin{bmatrix} 1 & 0 & -S_\theta \\ 0 & C_\phi & C_\theta S_\phi \\ 0 & -S_\phi & C_\theta C_\phi \end{bmatrix} \quad (6)$$

with the condition that Wη can be inverted whenever $\theta \neq (2k-1)\pi/2$, ($k \in Z$), its inverse will be used since these values are not reached during maneuvers with an autonomous underwater vehicle. pitch angle values. Thus the transformation matrix for angular velocities from the inertial to the body frame is Wη and from the body to the inertial frame is Wη -1.

Then expressing the matrix:

$$W\eta^{-1} = \begin{bmatrix} 1 & S_\phi T_\theta & C_\phi T_\theta \\ 0 & C_\phi & -S_\phi \\ 0 & S_\phi / C_\theta & C_\phi / C_\theta \end{bmatrix} \quad (7)$$

Assigning: $J_1(\eta)=R$ y $J_2(\eta) = W\eta^{-1}$, can be expressed $J(\eta)$ as indicated in the equation (6).

$$J(\eta) = \begin{bmatrix} J_1(\eta) & 0 \\ 0 & J_2(\eta) \end{bmatrix} \quad (8)$$

$$J(\eta) = \begin{bmatrix} C_\psi C_\theta & C_\psi S_\theta S_\phi - S_\psi C_\phi & C_\psi S_\theta C_\phi + S_\psi S_\phi & 0 & 0 & 0 \\ S_\psi C_\theta & S_\psi S_\theta S_\phi + C_\psi C_\phi & S_\psi S_\theta C_\phi - C_\psi S_\phi & 0 & 0 & 0 \\ -S_\theta & C_\theta S_\phi & C_\theta C_\phi & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & S_\phi t_\theta & C_\phi t_\theta \\ 0 & 0 & 0 & 0 & C_\phi & -S_\phi \\ 0 & 0 & 0 & 0 & \frac{S_\phi}{C_\theta} & \frac{C_\phi}{C_\theta} \end{bmatrix} \quad (9)$$

2.3 Linear velocity of the AUV.

Remembering that the coordinates of the center of mass of the vehicle with respect to the inertial reference system are given by the vector η (1):

$$\eta = \begin{bmatrix}\eta_1\\ \eta_2\end{bmatrix} \qquad con\ \eta_1 = [x,y,z]^T \qquad y\ \ \eta_2 = [\varphi,\ \theta,\ \psi]^T$$

The time derivative of η1 is the speed of the center of mass of the vehicle (origin of the mobile reference system) expressed about the inertial reference axis.

Being υ_1 the speed of the center of mass of the vehicle OA (origin of the mobile coordinate system) with respect to the origin of the inertial reference system, expressed with respect to the mobile reference axis, the conversion between and $\dot{\eta}_1$ y υ_1 can be established by means of the transformation matrix J1(η), so that the linear velocity of the vehicle measured in coordinates of the inertial frame, can be expressed as the linear velocity of the vehicle in coordinates of the body frame.

$$\dot{\eta}_1 = J_1(\eta)\,\upsilon_1 \quad \text{con} \quad \upsilon_1 = [u \quad v \quad w]^T \tag{10}$$

Then, with *J1(η)=R*, replacing in equation (10), by the rotation matrix R, the components of the AUV velocity in inertial coordinates are obtained, as:

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\end{bmatrix} = \begin{bmatrix} c(\psi)\,c(\theta) & c(\psi)s(\phi)s(\theta)-c(\phi)s(\psi) & s(\phi)s(\psi)+c(\phi)c(\psi)s(\theta)\\ c(\theta)s(\psi & c(\phi)c(\psi)+s(\phi)s(\psi)s(\theta) & c(\phi)s(\psi)s(\theta)-c(\psi)s(\phi)\\ -s(\theta) & c(\theta)s(\phi) & c(\phi)c(\theta) \end{bmatrix} \begin{bmatrix}u\\ v\\ w\end{bmatrix} \tag{11}$$

2.4 Angular velocity of the AUV

The time derivative of η_2, where $\eta_2 = [\varphi,\ \theta,\ \psi]$ T, determines the angular velocity of the center of mass of the AUV, with respect to the inertial frame, expressed with respect to the fixed reference axis. Remembering that the angular velocities of the mobile reference system with respect to the inertial system, referred to the mobile reference system, are given by $\omega = [p \quad q \quad r]^T$, and that the relationship between the inertial and mobile reference systems is established through the transformation *J2 (η),* with $J2(\eta) = W\eta^{-1}$, and using equation (7), it is possible to express the variation in time of the angles ϕ, θ and ψ.

$$\begin{bmatrix}\dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix} = \begin{bmatrix}1 & S_\phi T_\theta & C_\phi T_\theta\\ 0 & C_\phi & -S_\phi\\ 0 & S_\phi/C_\theta & C_\phi/C_\theta\end{bmatrix}\begin{bmatrix}p\\ q\\ r\end{bmatrix} \tag{12}$$

Since it is a discontinuous function, this result will be valid for any angle θ, such that:
$\theta \neq (2k-1)\pi/2,\ (k \in Z)$.

$$\begin{aligned}\dot{\phi} &= p + q\ \text{sen}\phi\ \tan\theta + r\ \cos\phi\ \tan\theta\\ \dot{\theta} &= q\ \cos\phi - r\ \text{sen}\phi\\ \dot{\psi} &= q\ \text{sen}\phi\ \sec\theta + r\ \cos\phi\ \sec\theta\end{aligned} \tag{13}$$

The derivatives $(\dot{\phi}, \dot{\theta}, \dot{\psi})$ are distinct from the AUV angular velocities in the rigid body coordinate system (p, q, r). To obtain the relationship between the angular velocities in the system of axes attached to the leather, with the variation in time of the angles of ϕ, θ and ψ, the previous matrix given in equation (12) is inverted, so that the Rotational motion of the AUV is defined by the components of the angular velocities in the three axes: roll angular rate (p), pitch angular rate (q), and yaw angular rate (r), about the x_A, y_A, and axes. z_A respectively as:

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 1 & 0 & -S_\theta \\ 0 & C_\phi & C_\theta S_\phi \\ 0 & -S_\phi & C_\theta C_\phi \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \tag{14}$$

Note that for small angles p, q, r are approximately equal to $\dot{\phi}, \dot{\theta}, \dot{\psi}$, which is observed if the previous equations for small angles are solved. Thus, the resulting matrices expressed according to equation (6), can be expressed as:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\theta} \\ \dot{\phi} \\ \dot{\psi} \end{bmatrix} = [J(\eta)] \begin{bmatrix} u \\ v \\ w \\ p \\ q \\ r \end{bmatrix} \tag{15}$$

$$J(\eta) = \begin{bmatrix} C_\psi C_\theta & C_\psi S_\theta S_\phi - S_\psi C_\phi & C_\psi S_\theta C_\phi + S_\psi S_\phi & 0 & 0 & 0 \\ S_\psi C_\theta & S_\psi S_\theta S_\phi + C_\psi C_\phi & S_\psi S_\theta C_\phi - C_\psi S_\phi & 0 & 0 & 0 \\ -S_\theta & C_\theta S_\phi & C_\theta C_\phi & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & S_\phi t_\theta & C_\phi t_\theta \\ 0 & 0 & 0 & 0 & C_\phi & -S_\phi \\ 0 & 0 & 0 & 0 & \frac{S_\phi}{C_\theta} & \frac{C_\phi}{C_\theta} \end{bmatrix}$$

Summarizing, the speeds of the AUV with respect to the inertial axes can be expressed as:

$$\dot{x} = u[c(\psi)c(\theta)] - v[c(\phi)s(\psi) - c(\psi)s(\phi)s(\theta)] + w[s(\phi)s(\psi) + c(\phi)c(\psi)s(\theta)]$$
$$\dot{y} = u[c(\theta)s(\psi)] + v[c(\phi)c(\psi) + s(\phi)s(\psi)s(\theta)] - w[s(\phi)c(\psi) - c(\phi)s(\psi)s(\theta)]$$
$$\dot{z} = -u[s(\theta)] + v[c(\theta)s(\phi)] + w[c(\phi)c(\theta)]$$
$$\dot{\phi} = p + q\ sen\phi\ tan\theta + r\ cos\phi\ tan\theta$$
$$\dot{\theta} = q\ cos\phi - r\ sen\phi$$
$$\dot{\psi} = q\ sen\phi\ sec\theta + r\ cos\phi\ sec\theta$$

This also allows us to express the linear and angular velocities in the three axes:

$$u = \dot{x}[c(\psi)c(\theta)] + \dot{y}[c(\phi)s(\psi)] - \dot{z}\,[s(\theta)]$$
$$v = \dot{x}\ [c(\psi)s(\phi)s(\theta) - c(\phi)s(\psi)] + \dot{y}\ [c(\psi)c(\phi) + s(\psi)s(\phi)s(\theta)]\ \ \dot{z}\ [s(\phi)c(\theta)]$$
$$w = \dot{x}\ [s(\psi)s(\phi) + c(\psi)c(\phi)s(\theta) + \dot{y}\ [s(\psi)c(\phi)s(\theta) - c(\psi)s(\phi) + \dot{z}[c(\phi)c(\theta)]$$
$$p = \dot{\phi} - \dot{\psi}[s(\theta)]$$
$$q = \dot{\theta}[c(\phi)] + \dot{\psi}[c(\theta)s(\phi)]$$
$$r = -\dot{\theta}[s(\phi)] + \dot{\psi}[c(\phi)c(\theta)]$$

In the case of having the data of the speed of the current with respect to the system fixed to the AUV, the speed of the current with respect to the ground can be expressed, according to equation (3), as:

$$\upsilon_c = J(\eta)^{-1} (\eta)\, \dot{\eta}_c$$

3.- Dynamics of Underwater Vehicles

The dynamic model of an AUV describes the relationship between the movements of the vehicle and the forces exerted on it.

In this way, the external forces necessary for the vehicle to move in a certain way can be calculated, or on the contrary, the movement generated by the external forces to which the AUV is subject can be determined.

To obtain the equations of motion, it is assumed that the vehicle is a rigid body and that the fixed reference system to Earth is inertial. The first of this assumption allows not to take into account, for this analysis, the forces that act in a specific way between the mass elements, while the second eliminates the forces caused by the relative movement of the Earth in space [4].

The forces and moments to which the AUV is subjected, considering it as a rigid body, will be called as: X, forces along the x axis, Y, forces along the y axis, Z, forces along the z axis, K, Moments in the x axis, M, Moments in the y axis, N, Moments in the z axis

3.1 Equations of motion

The equations that represent the movement of a body in a three-dimensional space can be obtained from the conservation laws of the linear and angular moments of the mobile referred to an inertial reference system.

The resultant moment with respect to the center of mass of the rigid body of all the forces exerted on it (kinetic momentum theorem) [7] is:

$$\sum \vec{F} = \frac{d(\vec{G})}{d(t)} \quad ; \tag{16}$$

With $\vec{G}$ momentum of the system , $\vec{G} = \sum m_i \vec{v_i}$.

Considering the constant mass of the AUV, the sum of forces is expressed as:

$$\sum \vec{F} = m \frac{d(\vec{v})}{d(t)} \tag{17}$$

where $\sum \vec{F}$ is the sum of external forces on the system applied at the center of mass of the body. The derivative is performed with respect to a system of inertial axes. It is an absolute derivative. The velocity vector is also an absolute vector, which may be projected onto the desired reference system, either absolute or relative.

The resulting moment $\sum \vec{M_c}$ about the center of mass of the rigid body of all the forces exerted on it (Kinetic Momentum Theorem) [7] is:

$$\vec{M}_C = \frac{d(\vec{H})}{d(t)} \text{con} \quad \vec{H} = I_C \vec{\omega} \qquad \text{y} \quad \vec{\omega} = [p, q, r]^{\mathrm{T}} \tag{18}$$

Where $\vec{M}c$ is the moment of the forces around the center of mass, $\vec{H}$ is the kinetic moment about said center of mass, its derivative is also absolute, and $I_C$is the inertia matrix of the vehicle.

The inertia matrix $I_C \in R^{3x3}$ for a rigid body with respect to its center of mass is defined as:

$$I_C = \begin{bmatrix} I_x & -I_{yx} & -I_{xz} \\ -I_{yx} & I_y & -I_{yz} \\ -I_{zx} & -I_{zy} & I_z \end{bmatrix}, \qquad I_C = I_C^T > 0 \tag{19}$$

where Ix , Iy , and Iz are the moments of inertia with respect to the axes attached to the body and the products of inertia Ixy = Iyx , Ixz = Izx , Iyz = Izy .

$$I_x = \int_V (y^2 + z^2)\ \rho_m\, dV\ ;$$
$$I_y = \int_V (x^2 + z^2)\ \rho_m\, dV\ ;$$
$$I_z = \int_V (x^2 + y^2)\ \rho_m\, dV\ ;$$
$$I_{xy} = \int_V xy\ \ \rho_m\, dV = \int_V xy\ \ \rho_m\, dV = I_{yx}$$
$$I_{xz} = \int_V xz\ \ \rho_m\, dV = \int_V zx\ \ \rho_m\, dV = I_{zx}$$
$$I_{yz} = \int_V yz\ \rho_m\, dV = \int_V zy\ \ \rho_m\, dV = I_{zy}$$

To establish the equations of movement of the rigid body, the position vector is defined, of the center of mass (OA) of the body, with respect to the axes fixed to the ground considered inertial, as $\vec{r}_{G/OT}$. *According to Figure 2*

$$\vec{r}_{G/OT} = \vec{r}_{OA/OT} + \vec{r}_{G/OA} \tag{20}$$

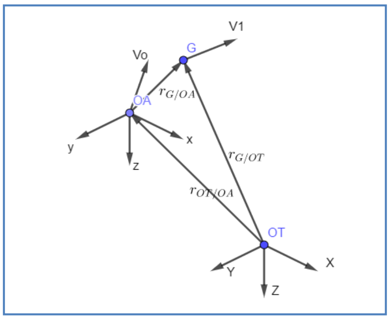

Figure 2

Deriving equation (20) with respect to time, we obtain:

$\frac{d}{dt}\vec{r}_{G/OT} = \dot{\vec{r}}_{OA/OT} + \left(\frac{d}{dt}\vec{r}_{G/OA} + \vec{\omega}\wedge\vec{r}_{G/OA}\right)$, con $\frac{d}{dt}\vec{r}_{G/OA} = 0$ y,

$$\vec{v}_{G/OT} = \vec{v}_{OA/OT} + \vec{\omega}\wedge\vec{r}_{G/OA}$$

Then: $\sum\vec{F} = m\frac{d(\vec{v})}{d(t)}$

$$F_G = m\left(\dot{\vec{v}}_{G/OT} + \vec{\omega}\wedge\vec{v}_{G/OA}\right)$$

$F_G = m\left[\dot{v}_{G/OA} + S(\omega)\,v_{G/OT}\right]$, con $S(\omega)v_{G/OT} = \omega\wedge v_{G/OT}$ (21)

If the rotation of the vehicle and the moments to which it is subjected are analyzed using the kinetic momentum theorem, it is observed that the resulting moment$\sum\overrightarrow{M_G}$ with respect to the center of mass (OA) of the vehicle, of all the forces that are exerted on it, is:

$$\overrightarrow{\boldsymbol{M}}_G = \frac{d(\vec{H})}{d(t)} = \frac{d}{dt}(I_G\,\vec{\omega})$$
$$= \frac{d}{dt}(I_G\,\vec{\omega}) + \vec{\omega}\wedge(I_G\,\vec{\omega})$$
$$= \left(I_G\,\dot{\vec{\omega}}\right) - (I_G\,\vec{\omega})\wedge\vec{\omega}$$

which can be expressed, : $S(I_G\omega)\,\omega = (I_G\omega)\wedge\omega$

$$I_G\,\dot{\omega} - S(I_G\omega)\,\omega = M_G$$

The Newton-Euler equations can be written in matrix form as:

$$M_{RA}\begin{bmatrix}\dot{v}_{\boldsymbol{G/OT}}\\ \dot{w}_b\end{bmatrix} + C_{RA}\begin{bmatrix}v_{\boldsymbol{G/OT}}\\ w_b\end{bmatrix} = \begin{bmatrix}F_G\\ \boldsymbol{M}_G\end{bmatrix}$$

$$\begin{bmatrix}mI_{3*3} & 0_{3*3}\\ 0_{3*3} & I_G\end{bmatrix}\begin{bmatrix}\dot{v}_{\boldsymbol{G/OT}}\\ \dot{w}_b\end{bmatrix} + \begin{bmatrix}mSw_b & 0_{3*3}\\ 0_{3*3} & -S(I_Gw_b)\end{bmatrix}\begin{bmatrix}v_{\boldsymbol{G/OT}}\\ \dot{w}_b\end{bmatrix} = \begin{bmatrix}F_G\\ \boldsymbol{M}_G\end{bmatrix} \quad (22)$$

The inertia matrix M_{RA} is expressed in the following form:

$$M_{RA} = \begin{bmatrix}mI_{3x3} & -mS(r_G)\\ mS(r_G) & I_G\end{bmatrix} \quad (23)$$

where m is the mass of the AUV, IG the inertia matrix of the vehicle, I3x3 the moments of inertia about the principal axes of inertia, $r_G = [x_G, y_G, z_G]^T$ the vector that determines the location of the origin (OA) with respect to the center of gravity of the vehicle and S(λ) is the symmetric matrix, such that:

$$S(\lambda) = \begin{bmatrix}0 & -\lambda_3 & \lambda_2\\ \lambda_3 & 0 & -\lambda_1\\ -\lambda_2 & \lambda_1 & 0\end{bmatrix}, \quad S(\lambda) = -S^T(\lambda) \quad (24)$$

With which the mass matrix M_{RA} is determined by:

$$M_{RA} = \begin{bmatrix} m & 0 & 0 & 0 & mz_G & -my_G \\ 0 & m & 0 & -mz_G & 0 & mx_G \\ 0 & 0 & m & my_G & -mx_G & 0 \\ 0 & -mz_G & my_G & I_x & -I_{xy} & -I_{xz} \\ mz_G & 0 & -mx_G & -I_{yx} & I_y & -I_{yz} \\ -my_G & mx_G & 0 & -I_{zx} & -I_{zy} & I_z \end{bmatrix} \tag{25}$$

Then the rigid body Coriolis matrix (C_{RA}), can be expressed as:

$$C_{RA} = \begin{bmatrix} 0_{3*3} & -mS(v) \\ -mS(v) & -S(Iw) \end{bmatrix} \tag{26}$$

$$C_{RA}(v) = \begin{bmatrix} 0_3 & -S(M_{R11}v_1 + M_{R12}v_2) \\ -S(M_{R11}v_1 + M_{R12}v_2) & -S(M_{R21}v_1 + M_{R22}v_2) \end{bmatrix} \tag{27}$$

$$C_{RA}(\upsilon) = \begin{bmatrix} 0 & 0 & 0 & m(y_G q + z_G r) & -m(x_G q - w) & -m(x_G r + v) \\ 0 & 0 & 0 & -m(y_G p + w) & m(z_G r + x_G p) & -m(y_G r - u) \\ 0 & 0 & 0 & m(v - z_G p) & -mu - mz_G q & m(x_G p + y_G q) \\ -m(y_G q + z_G r) & m(y_G p + w) & -mv + mz_G p & 0 & -I_{yz}q_{xz} - I_{xz}p + I_{zz}r & yp + I_{yz}r - I_{yy}q \\ m(x_G q - w) & -m(z_G r + x_G p) & mu + mz_G q & I_{yz}q + I_{xz}p - I_{zz}r & 0 & -I_{xy}q + I_{xx}p - I_{xz}r \\ m(x_G r + v) & -m(y_G r - u) & -m(x_G p + y_G q) & -I_{yz}r - I_{xy}p - I_{yy}q & I_{xy}q - I_{xx}p + I_{xz}r & 0 \end{bmatrix}$$
(28)

Finally, the equations of motion of a rigid body moving in space with respect to a system of inertial axes are obtained, as:

$$\begin{aligned} X &= m[\dot{u} - vr + wq - x_g(q^2 + r^2) + y_g(pq - \dot{r}) + z_g(pr + \dot{q})] \\ Y &= m[\dot{v} - wp + ur - y_g(p^2 + r^2) + z_g(rq - \dot{p}) + x_g(p + \dot{r})] \\ Z &= m[\dot{w} - uq + vp - z_g(p^2 + q^2) + x_g(pr - \dot{q}) + y_g(qr + \dot{p})] \\ K &= I_{xx}\dot{p} + (I_{zz} - I_{yy})qr - (\dot{r} + pq)I_{xz} + (r^2 - q^2)I_{yz} + (pr - \dot{q})I_{xy} \\ &\quad + m[y_g(\dot{w} - uq + vp) - z_g(\dot{v} - wp + ur)] \\ M &= I_{yy}\dot{q} + (I_{xx} - I_{zz})pr - (\dot{p} + rq)I_{xy} + (p^2 - r^2)I_{zx} + (pq - \dot{r})I_{yz} \\ &\quad + m[z_g(\dot{u} - vr + wp) - x_g(\dot{w} - uq + vp)] \\ N &= I_{zz}\dot{r} + (I_{yy} - I_{xx})qp - (\dot{q} + pr)I_{yz} + (q^2 - p^2)I_{xy} + (qr - \dot{p})I_{zx} \\ &\quad + m[x_g(\dot{v} - wp + ur) - y_g(\dot{u} - vr + wq)] \end{aligned}$$

Generalizing these equations for a point O of joint axes to the body that does not coincide with the center of mass of the rigid body and performing the corresponding calculations, the motion equations can be written in matrix form as expressed in equation (29):

$$\begin{bmatrix} F_0 \\ \boldsymbol{M}_0 \end{bmatrix} = M_R \begin{bmatrix} \dot{v}_0 \\ \dot{w}_b \end{bmatrix} + C_R\,(v_0\,, w_b) \begin{bmatrix} v_0 \\ w_b \end{bmatrix} \tag{29}$$

3.2 General movement of an AUV

Below are presented, from a Lagrangian approach, the equations of motion for bodies submerged in water. The movement of an AUV is the result of the action of the forces acting on it in a viscous fluid medium. These forces are mainly inertial forces, hydrodynamics and restorative forces.

3.2.1 Euler-Lagrange equations.

The description of vehicle dynamics with six degrees of freedom is commonly developed from a Lagrangian approach. Therefore, the kinetic energy and potential energy, called T and V respectively, are considered for the deduction of the equations of motion of the AUV, with respect to the inertial system. The application of Lagrange mechanics gives rise to n differential equations corresponding to n generalized coordinates (x, θ, φ, etc).

The Lagrangian *L* is the sum of the translational kinetic energies E_{trans} and the rotational energy E_{rot} minus the potential energy E_{pot} :

$$\boldsymbol{L} = \boldsymbol{T} - \boldsymbol{V} \tag{30}$$

$$L(\eta, \dot{\eta}) = E_{trans} + E_{rot} - \text{V} \quad \text{con} \quad T = E_{trans} + E_{rot} \;, \tag{31}$$

The equations that represent the movement of the vehicle in a three-dimensional space can be obtained from the laws of conservation of linear and angular momenta referred to an inertial reference system as previously developed [4]. Newton's second law can be expressed using the Lagrangian, for any coordinate system fixed to the body as:

$$M_{RA}\,\dot{v} + C_{RA}(v)v + M_A\,\dot{v} + C_A\,v + D(v)v + g(\eta) = \tau_{RA} \tag{32}$$

Where, M_{RA} is the inertia matrix determined from the configuration of symmetry of the AUV, considering its structure similar to an elongated ellipsoid with uniform mass distribution, CRA represents the Coriolis matrix, M_A is the inertia matrix of the added mass, C_A is the Coriolis matrix including the added mass, *D(v)* the damping matrix and the vector g(η) represents the restoring forces, composed of the force of gravity and the buoyant force. With $\tau_{RA} = \tau_{dh} + \tau_m + \tau_P$; where τ_{dh} y $\tau_{sh}$and are the moments generated by the hydrodynamic forces, the moments generated by the effects of wind and waves, and the torques produced by the propellers or any other force exerted on the AUV.

The velocity vector υ is the generalized velocity $v = [u, v, w, p, q, r]^T$where u, v, w are the linear components of pitch, roll, and roll and p, q, r are the angular components of roll, pitch, and yaw. generalized moments: $\tau_i = [X_i, Y_i, Z_i, K_i, M_i, N_i]^{\ T} \quad i = \in (hd, m, p)$

It is observed that equation (32) can be obtained by applying the Lagrangian equation,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\eta}}\right) - \frac{\partial L}{\partial \eta} = \tau_{RA}\,, \quad L = T - V \quad \text{and} \quad T = T_{RA} + T_A = \frac{1}{2}\,\dot{\eta}^T M(\eta)\,\dot{\eta}$$

where $M = M_{RA} + M_A$ (the inertia matrix including the added mass matrix)

$$\frac{\partial T}{\partial \eta} = \frac{1}{2}\dot{\eta}^T \frac{\partial M(\eta)}{\partial \eta}\dot{\eta}, \qquad \frac{\partial V}{\partial \eta} = g(\eta)$$

The vector *g(η)* represents the restoring forces (composed of the force of gravity and the force of buoyancy). Then:

$$\frac{\partial L}{\partial \eta} = \frac{\partial T}{\partial \eta} - \frac{\partial V}{\partial \eta} = \frac{1}{2}\,\dot{\eta}^T\,\frac{\partial M(\eta)}{\partial \eta}\,\dot{\eta} - g(\eta)$$

$$\frac{\partial L}{\partial \dot{\eta}} = M(\eta)\dot{\eta} - \frac{\partial V}{\partial \dot{\eta}} = M(\eta)\dot{\eta}\,, \quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\eta}}\right) = M(\eta)\ddot{\eta} + \dot{M}(\eta)\dot{\eta}$$

$$\dot{M}(\eta) = \dot{\eta}^T \frac{\partial M(\eta)}{\partial \eta}$$

Replacing $M(\eta)\ddot{\eta} + \frac{1}{2}\,\dot{M}(\eta)\dot{\eta} + g(\eta) = \tau_{RA}$ (33)

Analyzing the moments of the different forces acting on the AUV detailed in the equation. $\tau_{RA} = \tau_{dh} + \tau_m + \tau_P$. Hydrodynamic moments are called τ_{dh}, generated by the drag forces that oppose the movement and act in the opposite direction to the movement of the AUV, generating the generalized drag matrix *D(v)*, so the energy dissipated in this case can be considered as:

$$\frac{\partial D_d}{\partial \dot{\eta}} = D(v,\eta)\dot{\eta} \tag{34}$$

Substituting equation (34) in equation (33), we obtain:

$$M(\eta)\ddot{\eta} + \frac{1}{2}\dot{M}(\eta)\dot{\eta} + D(v,\eta)\dot{\eta} + g(\eta) = \tau \tag{35}$$

where $\tau = [\tau_X\,, \tau_Y\,, \tau_Z\,, \tau_K\,, \tau_M\,, \tau_N\,]^T$ is the input vector representing the forces exerted by the thrusters (or any other force-generating element) on the AUV.

It is observed that the term $\frac{1}{2}\,\dot{M}(\eta)$ represents the matrix of centrifugal and Coriolis forces of the submerged rigid body and the added mass,

$$C(v,\eta) = \frac{1}{2}\,\dot{M}(\eta) \tag{36}$$

Thus the dynamic model of an underwater vehicle can be written in its compact form as shown below:

$$\boldsymbol{M}\,\dot{\boldsymbol{v}} + \boldsymbol{C}(\boldsymbol{v})\boldsymbol{v} + \boldsymbol{D}(\boldsymbol{v})\boldsymbol{v} + \boldsymbol{g}(\boldsymbol{\eta}) = \boldsymbol{\tau} \tag{37}$$

where: $M = M_{RA} + M_A$, is the inertia matrix including the added mass, and $C(v) = C_{RA} + C_A$ is the Coriolis matrix, including the added mass.

3.2.2 Added mass matrix

The apparent increase in the mass and in general of the inertial properties of a body immersed in a fluid is known as added mass. When a body moves in a fluid, a certain amount of fluid must move around it. When the body accelerates, then the fluid must also accelerate. Therefore, more force is required to accelerate the body in a fluid than in a vacuum. Since force is related to mass and acceleration, we can think of the additional force in terms of an imaginary addition of mass to the object in the fluid.

The added masses are the forces and moments induced by the pressure due to the accelerated motion of the body. These forces and moments are proportional to the acceleration of the vehicle [8]. In such a way that any movement of the AUV will cause a movement of the stationary fluid in the opposite direction. In completely submerged vehicles it is usually assumed that the added mass coefficients are constant [1]. In this case, the analysis is restricted to submerged vehicles that move at low speed and bodies with three planes of symmetry are considered, which allows not taking into account the elements of the MA matrix that are outside the main diagonal [9]. This facilitates the calculation of MA since elements that are not part of the main diagonal are very difficult to calculate, analytically.

These added mass coefficients are defined as the proportionality constants, which relate the linear and angular accelerations with each of the forces and hydrodynamic moments that they generate. Thus, the hydrodynamic force along the x-axis due to acceleration in the x-direction is expressed as:

$$X_A = -X_{\dot{u}}\dot{u} \quad \text{where} \quad X_{\dot{u}} = \frac{\partial X}{\partial \dot{u}} \tag{38}$$

Similarly, all other added mass coefficients can be defined for a vehicle whose acceleration components are $((\dot{u},\ \dot{v}\,, \dot{w}, \dot{p}\,, \dot{q}\,, \dot{r}))$. Conforming the added mass matrix as a square matrix of order 6:

$$M_A = -\begin{bmatrix} X_{\dot{u}} & X_{\dot{v}} & X_{\dot{w}} & X_{\dot{p}} & X_{\dot{q}} & X_{\dot{r}} \\ Y_{\dot{u}} & Y_{\dot{v}} & Y_{\dot{w}} & Y_{\dot{p}} & Y_{\dot{q}} & Y_{\dot{r}} \\ Z_{\dot{u}} & Z_{\dot{v}} & Z_{\dot{w}} & Z_{\dot{p}} & Z_{\dot{q}} & Z_{\dot{r}} \\ K_{\dot{u}} & K_{\dot{v}} & K_{\dot{w}} & K_{\dot{p}} & K_{\dot{q}} & K_{\dot{r}} \\ M_{\dot{u}} & M_{\dot{v}} & M_{\dot{w}} & M_{\dot{p}} & M_{\dot{q}} & M_{\dot{r}} \\ N_{\dot{u}} & N_{\dot{v}} & N_{\dot{w}} & N_{\dot{p}} & N_{\dot{q}} & N_{\dot{r}} \end{bmatrix} \tag{39}$$

This matrix can be expressed in terms of four submatrices:

$$M_A = \begin{bmatrix} M_{A11} & M_{A12} \\ M_{A21} & M_{A22} \end{bmatrix} \tag{40}$$

Finally the matrix $M = M_{RA} + M_A$ can be written as:

$$M = \begin{bmatrix} m + X_{\dot u} & X_{\dot v} & X_{\dot w} & X_{\dot p} & mz_c + X_{\dot q} & -my_c + X_{\dot r} \\ Y_{\dot u} & m + Y_{\dot v} & Y_{\dot w} & -mz_c + Y_{\dot p} & Y_{q} & mx_c + Y_{\dot r} \\ Z_{\dot u} & Z_{\dot v} & m + Z_{\dot w} & my_c + Z_{\dot p} & -mx_c + Z_{\dot q} & Z_{\dot r} \\ K_{\dot u} & -mz_c + K_{\dot v} & my_c + K_{\dot w} & I_x + K_{\dot p} & -I_{xy} + K_{\dot q} & I_{xz} + K_{\dot r} \\ mz_c + M_{\dot u} & M_{\dot v} & mx_c + M_{\dot w} & -I_{xy} + M_{\dot p} & I_y + M_{\dot q} & -I_{yz} + M_{\dot r} \\ -my_c + N_{\dot u} & mx_c + N_{\dot v} & N_{\dot w} & -I_{xy} + N_{\dot p} & -I_{yx} + N_{\dot q} & I_z + N_{\dot r} \end{bmatrix} \quad (41)$$

3.2.3 Centripetal and Corioli Forces

The Coriolis matrix, *C(v),* is composed of two components, the Coriolis and centripetal matrix of the rigid body, $C_{RA}(v)$, and the Coriolis matrix of the added mass, *CA(v).* It is given by the following expression:

$$C(v)=C_{RA}+C_A(v) \quad (42)$$

The Coriolis matrix relative to the hydrodynamic effects derived from the added mass to the system is calculated from the added mass matrix and the operator S(λ). If the symmetric matrix ASim, is considered, its product with the velocity vector υ determines the coefficients of S(λ).

$$A_{Sim} = \begin{bmatrix} X_{\dot u} & X_{\dot v} & X_{\dot w} & X_{\dot p} & X_{\dot q} & X_{\dot r} \\ Y_{\dot u} & Y_{\dot v} & Y_{\dot w} & Y_{\dot p} & Y_{\dot q} & Y_{\dot r} \\ Z_{\dot u} & Z_{\dot v} & Z_{\dot w} & Z_{\dot p} & Z_{\dot q} & Z_{\dot r} \\ K_{\dot u} & K_{\dot v} & K_{\dot w} & K_{\dot p} & K_{\dot q} & K_{\dot r} \\ M_{\dot u} & M_{\dot v} & M_{\dot w} & M_{\dot p} & M_{\dot q} & M_{\dot r} \\ N_{\dot u} & N_{\dot v} & N_{\dot w} & N_{\dot p} & N_{\dot q} & N_{\dot r} \end{bmatrix} \quad (43)$$

And the added mass Coriolis matrix:

$$C_A = \begin{bmatrix} 0_{3*3} & -S(A_{11}v + A_{12}w) \\ -S(A_{11}v + A_{12}w) & -S(A_{21}v + A_{22}w \end{bmatrix} \quad (44)$$

$$C_A\ (v) = \begin{bmatrix} 0 & 0 & 0 & 0 & -Z_{\dot w}w & Y_{\dot v}v \\ 0 & 0 & 0 & -Z_{\dot w}w & 0 & -X_{\dot u}u \\ 0 & 0 & 0 & -Y_{\dot v}v & X_{\dot u}u & 0 \\ 0 & -Z_{\dot w}w & Y_{\dot v}v & 0 & -N_{\dot r}r & M_{\dot q}q \\ Z_{\dot w}w & 0 & -X_{\dot u}u & N_{\dot r}r & 0 & -K_{\dot p}p \\ -Y_{\dot v}v & X_{\dot u}u & 0 & -M_{\dot q}q & K_{\dot p}p & 0 \end{bmatrix} \quad (45)$$

4.- Hydrodynamic damping.

Underwater vehicles are affected by hydrodynamic damping, which is caused by linear and quadratic friction due to the presence of laminar and turbulent flows, and by quadratic resistance [10]; [11].

The forces and moments related to damping are a function of the relative motion of the fluid. In the areas where AUVs usually operate, the flow is turbulent. Under these conditions the friction due to the drag force causes linear and quadratic effects. The total drag resistance is defined as the sum of its linear and quadratic components. On the one hand, the quadratic terms of the lift or lift force (DQ) and on the other hand, the linear terms of the friction force (DL) [4].

Given the low speeds of the AUV under study and its symmetry, a simplification in the parameters of the hydrodynamic damping force is proposed, which consists of taking the main diagonal of the matrix of linear terms and the matrix of quadratic terms.

$$D = D_L + D_Q(v) \tag{46}$$

Where, DL is a 6×6 matrix that groups the linear damping terms and DQ (v) includes the quadratic coefficients [9].

$$D_L = \{X_u, Y_v, Z_w, K_p, M_q, N_r\},\ D_Q = \{X_{u|u|}, Y_{v|v|}, Z_{w|w|}, K_{p|p|}, M_{q|q|}, N_{r|r|}\}$$

$$D(v) = \begin{bmatrix} X_u+X_{u|u|}|u| & 0 & 0 & 0 & 0 & 0 \\ 0 & Y_v+Y_{v|v|}|v| & 0 & 0 & 0 & 0 \\ 0 & 0 & Z_w+Z_{w|w|}|w| & 0 & 0 & 0 \\ 0 & 0 & 0 & K_p+K_{p|p|}|p| & 0 & 0 \\ 0 & 0 & 0 & 0 & M_q+M_{q|q|}|q| & 0 \\ 0 & 0 & 0 & 0 & 0 & N_r+N_{r|r|}|r| \end{bmatrix} \tag{47}$$

The elements that make up this diagonal structure can be determined from experiments [12]; [13]. on the case of determining the linear and quadratic terms in X.

$$X = -\left(\frac{1}{2}\rho C_d A_f\right) u|u| = X_{u|u|}|u| \tag{48}$$

where $$X_{u|u|} = \frac{\partial X}{\partial u|u|} = -\frac{1}{2}\rho C_d A_f \tag{49}$$

Where ρ is the density of the water, C_d is the coefficient of resistance and A_f is the surface area of the vehicle facing the flow.

4.1 Hydrostatic terms

In hydrodynamics, gravitational and buoyancy forces are known as restoring forces [9]. Gravitational forces act at the center of gravity of the vehicle, whose coordinates are defined by the vector $r_G = [x_G; y_G; z_G]^T$. On the other hand, in the center of buoyancy, defined by $r_A = [x_A; y_A; z_A]$, the buoyancy forces act. The weight of a submerged body can be determined as $W = mg$, where: (W) is the weight of the vehicle (N); m is the mass of the vehicle (kg) and g is the gravitational constant (m/s^2). The push or buoyancy force is defined as $E = \rho g V$, with: (E) is the push received by the body; (N) ρ is the density of the displaced fluid (kg/m^3): (g) is the gravitational constant (m/s^2) and (V) is the volume of displaced fluid (m^3). The distance between the (CG) and (CA) of the vehicle is defined by the vector:

$$r_G = [x_G; y_G; z_G]^T = [x_G - x_A; y_G - y_A; z_G - z_A]^T .$$

Then transferring the weight and the thrust of the body to the mobile reference system (OA):

$$f_W = R^T \begin{bmatrix} 0 \\ 0 \\ W \end{bmatrix}, \quad f_E = -R^T \begin{bmatrix} 0 \\ 0 \\ E \end{bmatrix} \tag{50}$$

$$\text{Con } R^T = \begin{bmatrix} C_\Psi C_\theta & S_\Psi C_\theta & -S_\theta \\ C_\Psi S_\theta S_\phi - S_\Psi C_\phi & S_\Psi S_\theta S_\phi + C_\Psi C_\phi & C_\theta S_\phi \\ C_\Psi S_\theta C_\phi + S_\Psi S_\phi & S_\Psi S_\theta C_\phi - C_\Psi S_\phi & C_\theta C_\phi \end{bmatrix}$$

$$f_W = \begin{bmatrix} C_\Psi C_\theta & S_\Psi C_\theta & -S_\theta \\ C_\Psi S_\theta S_\phi - S_\Psi C_\phi & S_\Psi S_\theta S_\phi + C_\Psi C_\phi & C_\theta S_\phi \\ C_\Psi S_\theta C_\phi + S_\Psi S_\phi & S_\Psi S_\theta C_\phi - C_\Psi S_\phi & C_\theta C_\phi \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ W \end{bmatrix}; \quad f_W = \begin{bmatrix} -S_\theta \mathrm{W} \\ C_\theta S_\phi \mathrm{W} \\ C_\theta C_\phi \end{bmatrix} \tag{51}$$

$$f_E = \begin{bmatrix} C_\Psi C_\theta & S_\Psi C_\theta & -S_\theta \\ C_\Psi S_\theta S_\phi - S_\Psi C_\phi & S_\Psi S_\theta S_\phi + C_\Psi C_\phi & C_\theta S_\phi \\ C_\Psi S_\theta C_\phi + S_\Psi S_\phi & S_\Psi S_\theta C_\phi - C_\Psi S_\phi & C_\theta C_\phi \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ E \end{bmatrix}; \quad f_E = \begin{bmatrix} -S_\theta \mathrm{E} \\ C_\theta S_\phi \mathrm{E} \\ C_\theta C_\phi \mathrm{E} \end{bmatrix} \tag{52}$$

The vector of gravitational forces is expressed as a function of the coordinate system (OA) located in the AUV, remembering that the positive z axis was considered in the direction of the earth's surface, it is expressed as:

$$g(\eta) = -\begin{bmatrix} f_W + f_E \\ r_G \times f_W + r_A \times f_E \end{bmatrix} \tag{53}$$

where r_G is the vector that relates the center of gravity to the moving reference axis and rA is the vector that relates the moving reference axis to the inertial one.
Replacing in equation (53) by the forces obtained in equations (51) and (52) the vector of gravitational forces is obtained, as:

$$g(\eta) = \begin{bmatrix} S_\theta(W - E) \\ C_\theta S_\phi(E - W) \\ C_\theta C_\phi(E - W) \\ (y_A E - y_G W)C_\theta C_\phi + (z_G W - z_A E)C_\theta S_\phi \\ (-x_A E + x_G W)C_\theta C_\phi + (z_G W - z_A E)S_\theta \\ (x_A E - x_G W)C_\theta S_\phi - (y_G W - y_A E)S_\theta \end{bmatrix} \tag{54}$$

5. - Description of the experimental vehicle

The model presented is an underwater robot that is structurally designed to be built using a 3D printer. The different watertight compartments that make it up allow the easy assembly of electronic devices and components, batteries, cameras, motors, etc. Which facilitates their exchange according to experimental needs.
As can be seen in Figure 3, it has a watertight vertical cylinder where the main components are located. It has five engines with their respective nozzles located on the periphery of the cylinder and a camera located at the front of the AUV. To improve the hydrodynamic behavior of the design, it is proposed to

provide the structure with a hemispherical casing as indicated in Figure 4. Its dimensions are 48 x 30 x 20 cm with an approximate weight of 10 kg. Mechanically it was designed to be stable.

Figure 3 shows a CAD design of the internal structure of the AUV and Figure 4 shows this same structure with its corresponding hull and defines the inertial reference system, with origin at the OT point, and the system of coordinates attached to the AUV, originating from a point belonging to the vehicle coinciding with its center of mass (OA) and A = [$\vec{\mathbf{x}}_A$,$\vec{\mathbf{y}}_A$, $\vec{\mathbf{z}}_A$] attached to the AUV, where x_A; y_A axes; and z_A are made to coincide with the axes of inertia of the AUV. The axis $\vec{\mathbf{x}}_A$ is taken to be coincident with the direction of advance of the AUV, $\vec{\mathbf{y}}_A$ is orthogonal to $\vec{\mathbf{x}}_A$, while $\vec{\mathbf{z}}_A$ is oriented downward and orthogonal to the $\vec{\mathbf{x}}_A$ $\vec{\mathbf{y}}_A$.plane.

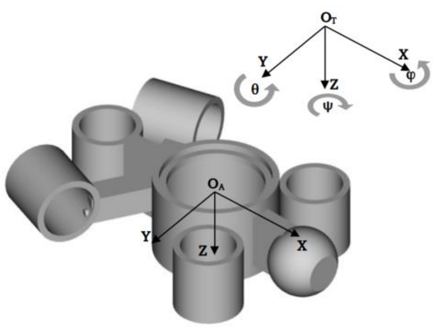

Figure 3

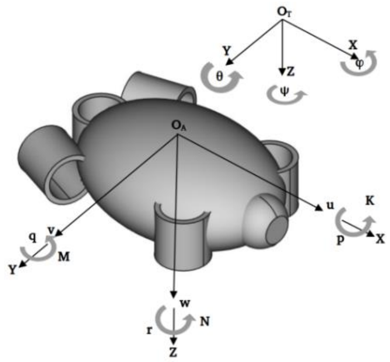

Figure 4

5.2 Description of the movement of the vehicle.

The vehicle is made up of five propellers that produce forces and torques, which are arranged on the central structure in the following way: three in a vertical position, two in front and one in the back, which are applied for movements of elevation, pitch and roll and the other two on the sides of the cylinder, one on the right side and one on the left side, to control the forward, yaw and roll movements. The angle between the longitudinal direction and the direction of the force of the lateral thrusters is 30°.

By design, the AUV is symmetrical in two of its axes and is mechanically stable in roll angle, which means that lateral displacement is small. It has five control inputs, where fi is the force of each thruster. The effect of these forces on the vehicle depends on their magnitude and their point of application.

As previously developed, the movement of the vehicle can be defined with the six components of position and attitude in the six degrees of freedom of the AUV, referred to an inertial frame, according to:

$$\begin{aligned} &\eta = [\eta_1{}^T, \eta_2{}^T]^T \quad ; \quad \eta_1 = [x,y,z]^T \quad ; \eta_2 = [\varphi, \theta, \psi]^T \\ &v = [v_1{}^T, v_2{}^T]^T \quad ; \quad v_1 = [u,v,w]^T \quad ; v_2 = [p, q, r]^T \\ &\tau = [\tau_1{}^T, \tau_2{}^T]^T \quad ; \quad \tau_1 = [X,Y,Z]^T \quad ; \tau_2 = [K, M, N]^T \end{aligned} \tag{55}$$

where η is the position and orientation vector with coordinates in the inertial reference system, v represents the linear and angular velocity in the frame fixed to the body and $\boldsymbol{\tau}$ the external forces and moments acting on the body.

The dynamic model of the AUV, as previously developed in section 3.2, can be represented from the following Newton-Euler equations of motion.

$$\boldsymbol{M\dot{v} + C(v)v + D(v)v + g(\eta) = \tau} \tag{56}$$

$$\boldsymbol{\dot{\eta} = J\eta(v)} \tag{57}$$

where, M represents the inertia matrix (including added mass), $\boldsymbol{C}(\boldsymbol{v})$ includes the terms Coriolis and centripetal force (including added mass), $\boldsymbol{D}(\boldsymbol{v})$describes the hydrodynamic damping matrix, $\boldsymbol{g}(\boldsymbol{\eta})$is the vector of gravitational force and buoyancy, represents the vector of control inputs and $J\eta$ is the kinematic transformation between the body and the inertial frame.

5.2.1 Inertia matrix

Some simplifications on the AUV dynamics are made, to facilitate the study of the model dynamics. It can be assumed that the AUV is symmetrical in the three planes of movement, since the vehicle works at a low speed, using 1m/sec as a maximum value. In the case of the experimental design, the AUV is symmetric with respect to the x-z plane and nearly symmetric with respect to the x-y plane. Although the AUV is not totally symmetric with respect to the y-z plane, due to its low speed, it is assumed that the vehicle is symmetric with respect to this plane, which allows decoupling the degrees of freedom. Likewise, the vehicle remains almost horizontal in all maneuvers and is stabilized, since the center of gravity and the center of buoyancy are correctly aligned. The Ixy crossed moments of inertia; Iyx; Iyz; Izy are negligible due to the symmetry of the AUV.

It is implemented that the thrust is slightly greater than the weight, this exerts an upward force of approximately 0.4% of the weight. This similarity between the forces is obtained since the model has been designed and the necessary auxiliary masses have been introduced for this to be true.

Table 2 presents the main parameters of the AUV, considering the geometry of the vehicle, its properties and the main characteristics of the materials to be used in the construction. To simplify the calculation of the inertia tensor, it is considered that the center of inertia coincides with the geometric center of the body.

Properties	**Values**	**Units**	**Symbols**
AUV Dimensions	48x24x12	m	*l x b x h*
AUV mass	10,23	Kg	*m*
Inertia tensor in x	0,052	$Kg.m^2$	I_{xx}
Inertia tensor in y	0,12	$Kg.m^2$	I_{yy}
Inertia tensor in z	0,16	$Kg.m^2$	I_{zz}

Table 2: Physical parameters of the AUV

Considering that the vehicle will move at very low speed, some damping parameters and added mass can be estimated using Solidworks.. With these considerations, the MRA matrix obtained in equations (25) can be written as:

$$M_{RA} = \begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & I_x & 0 & 0 \\ 0 & 0 & 0 & 0 & I_y & 0 \\ 0 & 0 & 0 & 0 & 0 & I_z \end{bmatrix} \quad (58)$$

$$M_{RA} = \begin{bmatrix} 10{,}23 & 0 & 0 & 0 & 0 & 0 \\ 0 & 10{,}23 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10{,}23 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0{,}052 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0{,}12 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0{,}16 \end{bmatrix} \quad (59)$$

5.2.2 Added mass

There is a vehicle that operates at low speed and that has three planes of symmetry, since the terms of the added mass matrix depend on the format of the AUV, these characteristics make it possible to not take into account the elements of the MA matrix, expressed according to the equation (39) that are outside the main diagonal [9].

So that the expressions to determine the matrices M_A and $C_A(v)$ can be simplified as:

$$M_A = -diag\{X_{\dot{u}}, Y_{\dot{v}}, Z_{\dot{w}}, K_{\dot{p}}, M_{\dot{q}}, N_{\dot{r}}\}, \quad (60)$$

There is a set of mathematical expressions that allow calculating the coefficients of the diagonal structure of M_A [1]. They are applicable to those vehicles whose geometric shape is similar to an elongated spheroid. The current design of the AUV under study allows an approximation to an elongated spheroid. Assuming that the vehicle under study has the shape of a prolate ellipsoid, and approximating its symmetry in the three spatial directions, the distance between the horizontal thrusters, perpendicular to the direction of advance in the x-axis, is taken as the dimension b in the x-direction. of the y-axis of the ellipsoid. Thus the principal semiaxis of the ellipsoid is the total length of the vehicle a and the secondary semiaxis has dimension b.

The added mass matrix parameters are constant when the vehicle is completely submerged. These parameters are generally in the neighborhood of 10% to 100% of the corresponding parameters in the rigid body mass matrix [14]. Values of $X\dot{u}$=-6.73kg ,$\dot{v}$=$-6.721kg$, $Z\dot{w}$= -5.56kg = -0.001kg.m2 and $M\dot{q}$=$N\dot{r}$= $-0.01220\ kg\ m2$ have been obtained. The results obtained are consistent, since they account for between 10% and 70% of the magnitude of the vehicle's mass, which coincides with other studies carried out on autonomous vehicles.

The inertia matrix, including the added masses for the AUV, is expressed as the sum of equations (58) and (60), as can be seen in equation (61).

$$M = M_{RA} + M_A = \begin{bmatrix} 16{,}96 & 0 & 0 & 0 & 0 & 0 \\ 0 & 16{,}95 & 0 & 0 & 0 & 0 \\ 0 & 0 & 15{,}79 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0{,}053 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0{,}132 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0{,}172 \end{bmatrix} \quad (61)$$

5.2.3 Coriolis matrix

The Coriolis and centripetal matrix of the rigid body, $C_{RA}(v)$ is obtained by substituting in equation (28) the values of the vehicle under study.

$$C_{RA}(\upsilon) = \begin{bmatrix} 0 & 0 & 0 & 0 & mw & 0-mv \\ 0 & 0 & 0 & -mw & 0 & mu \\ 0 & 0 & 0 & mv & -mu & 0 \\ 0 & mw & -mv & 0 & I_{zz}r & -I_{yy}q \\ mw & 0 & mu & -I_{zz}r & 0 & I_{xx}p \\ mv & -mu & 0 & I_{yy}q & -I_{xx}p & 0 \end{bmatrix} \tag{62}$$

$$C_{RA}(\upsilon) = \begin{bmatrix} 0 & 0 & 0 & 0 & 10{,}23w & -10{,}23v \\ 0 & 0 & 0 & -10{,}23w & 0 & 10{,}23u \\ 0 & 0 & 0 & 10{,}23v & -10{,}23u & 0 \\ 0 & 10{,}23w & -10{,}23v & 0 & 0{,}16r & -0{,}12q \\ 10{,}23w & 0 & 10{,}23u & -0{,}16r & 0 & 0{,}052p \\ 10{,}23v & -10{,}23u & 0 & 0{,}12q & -0{,}052p & 0 \end{bmatrix} \tag{63}$$

The Coriolis matrix, relative to the hydrodynamic effects derived from the mass added to the system, can be calculated from the previous matrix and from theoperator S(λ). If the symmetric matrix ASim is considered, according to equation (43), its product with the velocity vector υ determines the coefficients of S(λ). Taking the main diagonal of the added mass matrix, the added mass Coriolis matrix, substituting into equation (44), is expressed as:

$$C_A(v) = \begin{bmatrix} 0 & 0 & 0 & 0 & -Z_{\dot{w}}w & Y_{\dot{v}}v \\ 0 & 0 & 0 & -Z_{\dot{w}}w & 0 & -X_{\dot{u}}u \\ 0 & 0 & 0 & -Y_{\dot{v}}v & X_{\dot{u}}u & 0 \\ 0 & -Z_{\dot{w}}w & Y_{\dot{v}}v & 0 & -N_{\dot{r}}r & M_{\dot{q}}q \\ Z_{\dot{w}}w & 0 & -X_{\dot{u}}u & N_{\dot{r}}r & 0 & -K_{\dot{p}}p \\ -Y_{\dot{v}}v & X_{\dot{u}}u & 0 & -M_{\dot{q}}q & K_{\dot{p}}p & 0 \end{bmatrix} \tag{64}$$

$$C_A(v) = \begin{bmatrix} 0 & 0 & 0 & 0 & 5{,}56w & -6{,}721v \\ 0 & 0 & 0 & 5{,}56w & 0 & 6{,}73u \\ 0 & 0 & 0 & 6{,}721v & -6{,}73u & 0 \\ 0 & 5{,}56w & -6{,}721v & 0 & 0{,}0122r & -0{,}0122q \\ -5{,}56w & 0 & 6{,}73u & -0{,}0122r & 0 & 0{,}001p \\ 6{,}721v & -6{,}73u & 0 & 0{,}0122q & -0{,}001p & 0 \end{bmatrix} \tag{65}$$

Performing the sum of equations (63) and (65) the Coriolis matrix is obtained, including the Coriolis matrix of added mass, as $C(v) = CRA(v) + CA(v)$ results:

$$\boldsymbol{C(\upsilon)} = \begin{bmatrix} 0 & 0 & 0 & 0 & (m-Z_{\dot{w}})w & -(m+Y_{\dot{v}})v \\ 0 & 0 & 0 & -(m+Z_{\dot{w}})w & 0 & (m-X_{\dot{u}})u \\ 0 & 0 & 0 & (m+Y_{\dot{v}})v & -(m+X_{\dot{u}})u & 0 \\ 0 & (m-Z_{\dot{w}})w & -(m+Y_{\dot{v}})v+ & 0 & (I_{zz}-N_{\dot{r}})r & -(I_{yy}+M_{\dot{q}})q \\ -(m+Z_{\dot{w}})w & 0 & (m-X_{\dot{u}})u & -I_{zz}r & 0 & (I_{xx}-K_{\dot{p}})p \\ (m+Y_{\dot{v}})v & -(m+X_{\dot{u}})u & 0 & (I_{yy}-M_{\dot{q}})q & -(I_{xx}-K_{\dot{p}})p & 0 \end{bmatrix} \tag{66}$$

5.2.4 Hydrodynamic damping

Considering the existing symmetry in the AUV, it is possible to establish the following mathematical relationships between several of the parameters of the matrix D(υ) [1], [10]:

$$D_L = \{X_u, Y_v, Z_w, K_p, M_q, N_r\},\ D_Q = \{X_{u|u|}, Y_{v|v|}, Z_{w|w|}, K_{p|p|}, M_{q|q|}, N_{r|r|}\}$$

$$\boldsymbol{D(v)} = \begin{bmatrix} X_u+X_{u|u|}|u| & 0 & 0 & 0 & 0 & 0 \\ 0 & Y_v+Y_{v|v|}|v| & 0 & 0 & 0 & 0 \\ 0 & 0 & Z_w+Z_{w|w|}|w| & 0 & 0 & 0 \\ 0 & 0 & 0 & K_p+K_{p|p|}|p| & 0 & 0 \\ 0 & 0 & 0 & 0 & M_q+M_{q|q|}|q| & 0 \\ 0 & 0 & 0 & 0 & 0 & N_r+N_{r|r|}|r| \end{bmatrix} \quad (67)$$

The terms related to linear and quadratic hydrodynamic damping are related to the drag force of the vehicle. The drag forces are calculated for the three axes at various speeds, taking 1m/s as the maximum speed value, as well as the respective moments. The calculation is carried out with the use of specialized simulation software and through iteration the convergence of the values is obtained.

5.2.5 Hydrostatic Terms

The matrix that contemplates the gravitational effects is defined by equation (54), for the vehicle under study the following considerations are made. In the first instance, it will be considered that the weight of the vehicle is equal to the thrust: W = E, in order to simplify the calculations, although in later stages it is proposed to give the AUV a weight slightly less than the thrust in order to easily recover the vehicle in the event of a failure in the propellants. The coordinate system is located in the center of mass of the vehicle and the center of flotation is coincident in the z axis with the center of gravity, considering $z_G - z_A = AG_z$, then we consider the center of flotation as the origin of coordinates of the system local, for which the buoyant force will not produce moments, but the gravitational force will, for which substituting in equation (54), we obtain:

$$g(\eta) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ AG_z W C_\theta S_\phi \\ AG_z W S_\theta \\ 0 \end{bmatrix} \quad (68)$$

5.2.6 Propulsion Forces

The vector τ can be calculated from the orientation and position matrix L and the force vector of the thrusters U.

$$\tau = LU \quad (69) \quad ; \qquad U = \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{bmatrix} \qquad (70)$$

The L matrix is made up of six rows, to introduce the orientation unit vector (u) and the position vector (r) of each thruster, and five columns, as many as the number of thrusters.

$$\begin{bmatrix} u_1 & u_1 & \dots & u_N \\ r_1 & r_2 & \dots & r_N \end{bmatrix}$$

The matrix L, for a number of thrusters equal to five, and their location according to Figure 4, has the following form:

$$L = \begin{bmatrix} cos\alpha & cos\alpha & 0 & 0 & 0 \\ -sen\alpha & sen\alpha & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & l_4 & -l_4 & 0 \\ 0 & 0 & l_3 & l_3 & -l_5 \\ l_1 cos\alpha + l_2 sen\alpha & -l_1 cos\alpha - l_2 sen\alpha & 0 & 0 & 0 \end{bmatrix}, \qquad (71)$$

Figure 4 shows the numbering of the AUV engines and the distances in which the engines are separated from the center of gravity. These distances are: L_1= 14cm, L_2= 13cm, L_3= 13cm, L_4= 10cm and L_5= 13cm. The angle α= 30°.

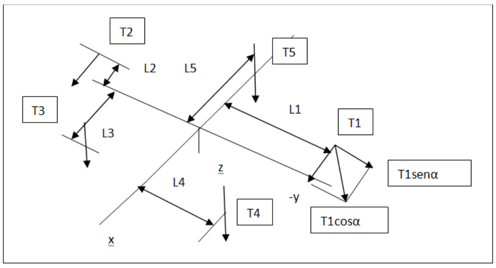

Figure 5

Substituting the previous values in equation (69) and performing the corresponding operations, the vector of forces and moments caused by the control inputs is obtained as expressed in equation (72).

$$\tau = \begin{bmatrix} \tau_x \\ \tau_y \\ \tau_z \\ \tau_K \\ \tau_M \\ \tau_N \end{bmatrix} = \begin{bmatrix} (T_1 + T_2) cos\alpha \\ (T_2 - T_1) sen\alpha \\ T_4 + T_3 + T_5 \\ (T_3 - T_4) l_4 \\ T_3 l_3 + T_4 l_3 - T_5 l_5 \\ (T_1 - T_2)(l_1 cos\alpha + l_2 sen\alpha) \end{bmatrix} \qquad (72)$$

As can be seen, all the matrix terms that make up the non-linear model, with six degrees of freedom (6DOF), of the presented design responding to vectorial equation (37) are thus defined.

Table 3 presents the main parameters of the vehicle considering the geometry of the AUV and that it will move at very low speed, calculated with the "SolidWork" program and the MATLAB "Simulink" tool.

Obtaining the 6 GDL model will allow, in subsequent studies, to synthesize the control loops that are desired to be implemented in the vehicle.

Parameters	**Values**	**Units**	**Description**
$X_{\dot{u}}$	-6,73	Kg	added mass
X_u	-10	Kg/s	linear damping
$X_{u\vee u\vee}$	-14,6	kg/m	axial drag
$Y_{\dot{v}}$	-6,72	Kg	added mass
Y_v	-12	Kg/s	linear damping
$Y_{v\vee v\vee}$	-16,6	kg/m	axial drag
$Z_{\dot{w}}$	-5,56	Kg	added mass
$Z_{w\vee w\vee}$	-19,6	kg/m	Cross Flow Drag
Z_w	-17	Kg/s	linear damping
$K_{\dot{p}}$	-0,001	$Kg.m^2$	added mass
K_p	-1,4	Kg/s	linear damping
$K_{p\vee p\vee}$	-1,15	kg/m	roll drag
$M_{\dot{q}}$	-0,012	$Kg.m^2$	added mass
M_q	-1,6	Kg/s	linear damping
$M_{q\vee q\vee}$	-1,19	kg/m	Cross Flow Drag
$N_{\dot{r}}$	-0,012	$Kg.m^2$	added mass
N_r	-1,6	Kg/s	linear damping
$N_{r\vee r\vee}$	-1,19	kg/m	Cross Flow Drag

Table 3: Hydrodynamic parameters

Counting on all the parameters of the 6 GDL model of the vehicle, its behavior can be analyzed through simulation using the TOOLBOX MSS tool belonging to MATLAB.

6. Results obtained

From a simulation with the MATLAB SIMULINK tool and using the parameters obtained in section 5, it was possible to visualize the dynamic characteristics of the AUV in a trajectory as shown in figure 6.

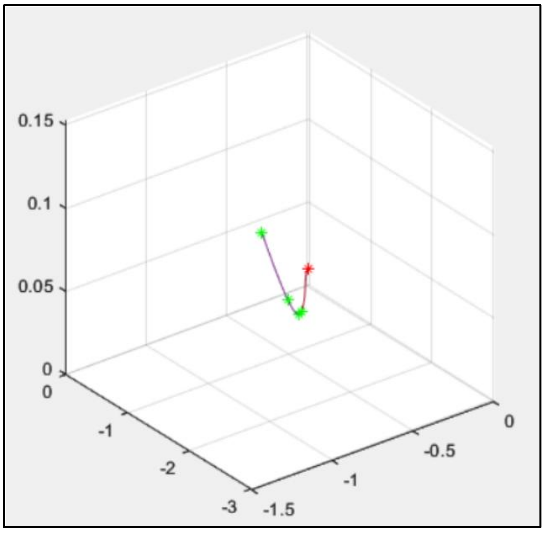

Figure 6. Simulated AUV trajectory

For this trajectory, the velocities of each coordinate axis were obtained.

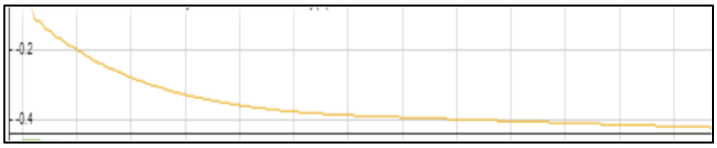

Figure 7. Speed about the axis(1)

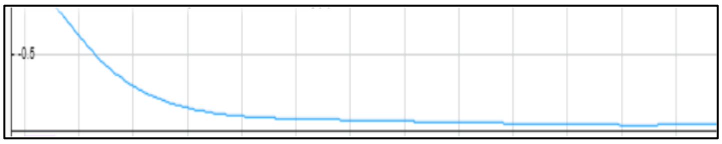

Figure 8. Speed about the axis(2)

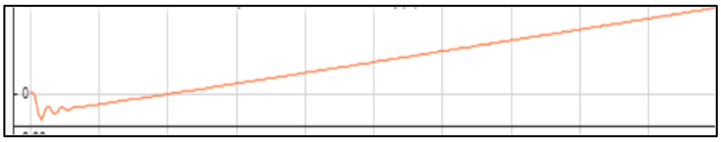

Figure 9. Speed about the axis (3)

The results obtained are consistent with works carried out on other AUVs with similar characteristics [11] and with other simulations [13].

7. Conclusions

The study of the dynamic characteristics of the proposed vehicle allowed obtaining parameters to study its behavior through simulations that can be used to analyze the behavior of the AUV under different operating conditions, and thus, evaluate its performance and/or propose improvements.

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